

Student Workbook

Graphs of Polynomial Functions

Learning Target: I can find the zeros and multiplicities of a polynomial and use them, together with end behavior, to describe or sketch its graph.

Vocabulary

Term	Meaning
Zero (Root)	A value of x where $f(x) = 0$; also an x -intercept of the graph.
Multiplicity	How many times a factor $(x-a)$ repeats in the factored polynomial.
Turning Point	A point where the graph changes from increasing to decreasing or vice versa.

Key Concept: Multiplicity Determines Crossing vs. Touching

THE RULE

Odd multiplicity (1, 3, 5, ...): the graph **CROSSES** the x -axis at that zero.

Even multiplicity (2, 4, 6, ...): the graph **TOUCHES** the x -axis and turns back (bounces).

EXAMPLE 1

$$f(x) = (x-2)(x+1)^2(x-4)^3$$

$x=2$: multiplicity _____ $\rightarrow$ _____

$x=-1$: multiplicity _____ $\rightarrow$ _____

$x=4$: multiplicity _____ $\rightarrow$ _____

Key Concept: Putting It All Together

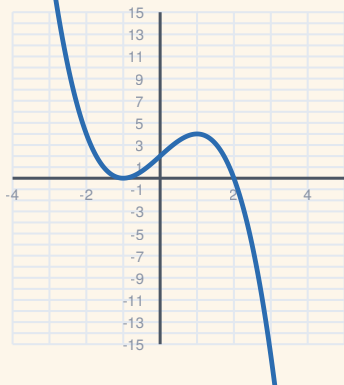
METHOD TO DESCRIBE A GRAPH 1. Find the zeros and their multiplicities (crosses or touches).

2. Find the degree and leading coefficient to determine end behavior.

3. The graph has **AT MOST** (degree - 1) turning points.

EXAMPLE 2

$$f(x) = -(x-2)(x+1)^2 \text{ (degree 3, negative leading$$

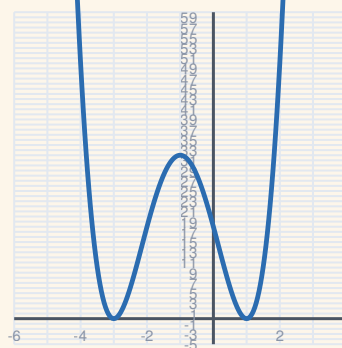


coeff)

Zeros: $x=2$ (crosses), $x=-1$ (touches). End behavior: left up, right down.

EXAMPLE 3

$$f(x) = 2(x+3)^2(x-1)^2 \text{ (degree 4, positive leading$$



coeff)

Zeros: $x=-3$ (touches), $x=1$ (touches). End behavior: both ends up.

COMMON MISTAKE

The multiplicity is the EXPONENT on the factor, not the zero's value! In $(x-4)^3$, the zero is $x=4$ with multiplicity 3 — don't mix these up.

Key Concept: Writing a Polynomial from Its Zeros**EXAMPLE 4**

Write a polynomial with zeros $x=-2$ (multiplicity 1), $x=3$ (multiplicity 2), and leading coefficient 1.

$f(x) = \underline{\hspace{2cm}}$

Guided Practice — Part A: Identify Zeros & Multiplicities

A1 $f(x) = (x-1)(x+3)^2(x-5)$

A2 $f(x) = x^2(x+2)^3(x-4)$

Guided Practice — Part B: Crosses or Touches?

B1 Using A1, state whether the graph crosses or touches at each zero.

B2 Using A2, state whether the graph crosses or touches at each zero.

Guided Practice — Part C: Describe the Full Graph

C1 $f(x) = -(x-2)(x+1)^2$

C2 $f(x) = 2(x+3)^2(x-1)^2$

Guided Practice — Part D: Write a Polynomial from Zeros

D1 Zeros $x=-1$ (mult. 1), $x=2$ (mult. 3), leading coefficient 1

D2 Zeros $x=0$ (mult. 2), $x=-4$ (mult. 1), leading coefficient -1

BEFORE YOU MOVE ON

Check with a partner: did you correctly match ODD multiplicity to "crosses" and EVEN multiplicity to "touches"?